

# The difficulty in the decay constants and spectra of $D_{sJ}^*$

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(Dated: February 1, 2008)

We examine the compatibility in predicted masses and the decay constants of  $D_{sJ}^*$  mesons in terms of two-quark contents. We find that the results of a specific model, which is governed by heavy quark limit, will encounter a challenge to fit both spectra and decay constants simultaneously.

In spite of the success of the quark model, the explanation with the same concept on light scalar mesons denoted by  $0^{++}$ , such as the nonet composed of isoscalars  $\sigma(600)$  and  $f_0(980)$ , isovector  $a_0(980)$  and isodoublet  $\kappa$ , still has puzzles of (a) why  $a_0(980)$  and  $f_0(980)$  are degenerate in masses and (b) why the widths of  $\sigma$  and  $\kappa$  are broader than those of  $a_0(980)$  and  $f_0(980)$  [1]. Probably, these scalar states consist of four rather than two quarks [2]. Moreover, the possibilities of  $K\bar{K}$  molecular states and scalar glueballs are also proposed. Hence, the conclusion is still uncertain.

The mysterious event happens not only in the light scalar mesons, but now also in the heavy charm-quark system. Recently, BABAR collaboration has observed one narrow state, denoted by  $D_{sJ}^*(2317)$ , from a  $D_s^+\pi^0$  mass distribution [3]. Afterward, the same state is confirmed by CLEO and a new state  $D_{sJ}^*(2460)$  is also seen in the  $D_s^{*+}\pi^0$  final state [4]. Finally, BELLE verifies the observations [5]. The two states  $D_{sJ}^*(2317)$  and  $D_{sJ}^*(2460)$  currently are identified as parity-even states with  $0^+$  and  $1^+$ , respectively. According to the experimental displays, the masses (widths) of both states are too low (narrow) and cannot match with theoretical predictions [6]. To explain the discrepancy, either the theoretical models have to be modified [7] or the observed states are the new composed states. To satisfy the latter, many interesting solutions have been suggested recently in Refs. [8, 9, 10, 11, 12, 13, 14].

It is interesting to speculate whether a two-quark picture is really enough to explain the spectra of the  $D_{sJ}^*$  system. Before examining the possibility, we first discuss the nonstrange parity-even states  $D^{**}$ , measured by CLEO [15] and BELLE [16] in  $B$  meson decays before the BABAR's observation. With the two-quark picture, there are four parity-even (angular momentum  $\ell = 1$ ) states described by  $J^P = 0^+, 1^+, 1^+$  and  $2^+$ , respectively. Here,  $J = j_q + S_Q$  is the total angular momentum of the corresponding meson and consists of the angular momentum of the light quark,  $j_q$ , and the spin of the heavy quark,  $S_Q$ , where  $j_q = S_q + \ell$  is combined by the spin and orbital angular momenta of the light quark. In the literature, they are usually labeled by  $D_0^*$ ,  $D'_1$ ,  $D_1$  and  $D_2^*$ , respectively. The first two belong to  $j_q = 1/2$ , while the last two are  $j_q = 3/2$ . In the heavy quark limit, it is known that  $D_{0(1)}^{*(*)}$  and  $D_{1(2)}^{(*)}$  belong to the doublets  $(0^+, 1^+)$  and  $(1^+, 2^+)$ , respectively [18]. Moreover, the former decay only via  $S$ -wave while the latter are through  $D$ -wave. Therefore, one expects that the widths of the former are much broader than those of the latter, which is consistent with the observations of CLEO and BELLE [15, 16]. Even the BELLE's updated data [17] also show the same phenomenon. We now summarize the results of CLEO and BELLE as follows. In CLEO, the masses (widths) of  $P$ -wave states are given by  $m_{D_1}(\Gamma_{D_1}) = 2422.0 \pm 2.1$  ( $18.9^{+4.6}_{-3.5}$ ) and  $m_{D_2^*}(\Gamma_{D_2^*}) = 2458.9 \pm 2.0$  ( $23 \pm 5$ ) MeV. In BELLE, the four states are all measured as  $m_{D_0^*}(\Gamma_{D_0^*}) = 2308 \pm 17 \pm 15 \pm 28$  ( $276 \pm 21 \pm 18 \pm 60$ ),  $m_{D'_1}(\Gamma_{D'_1}) = 2427.0 \pm 26 \pm 20 \pm 15$  ( $384^{+107}_{-75} \pm 24 \pm 70$ ),  $m_{D_1}(\Gamma_{D_1}) = 2421.4 \pm 1.5 \pm 0.4 \pm 0.8$  ( $23.7 \pm 2.7 \pm 0.2 \pm 4.0$ ), and  $m_{D_2^*}(\Gamma_{D_2^*}) = 2461.6 \pm 2.1 \pm 0.5 \pm 3.3$  ( $45.6 \pm 4.4 \pm 6.5 \pm 1.6$ ) MeV. Clearly, from BELLE's results, the predicted mass differences  $M(D_0^*) - M(D)$  [ $M(D'_1) - M(D^*)$ ]  $\sim 350$  MeV in the limit of the heavy quark symmetry [7], in which  $D(D^*)$  belong to the doublet  $(0^-, 1^-)$ , are smaller than experimental results  $\sim 420$  MeV. As mentioned in Ref. [7], the correction  $\Lambda_{QCD}/m_c$  will compensate for the shortage. Hence, if the parity-even nonstrange charm-meson obeys the chiral quark model, it is incomprehensible intuitively why the strange one cannot do.

In fact, there is somewhat a difference in the decays of  $D_{sJ}^*$  and  $D^{**}$ . Since the measured masses of  $D_{sJ}^*$  are just below the  $D^{(*)}K$  threshold and the corresponding widths are at the limit of the detector, being less than 10 MeV, both parity-even mesons could only decay through isospin violating channels to  $D\pi$  and  $D^*\pi$ . By this reason, one would understand why the widths of the parity-even mesons are so narrow. As a result, in terms of the pole of the Breit-Wigner formula, the effective masses of the propagating mesons while they are produced are estimated by  $M_{eff} \sim (M^2(D_{sJ}^*) + M(D_{sJ}^*)\Gamma(D_{sJ}^*))^{1/2} \approx M(D_{sJ}^*)$ . That is, there is no room in phase space for  $D_{sJ}^*$  decaying to the final states  $D^{(*)}K$ . The states in the  $\bar{c}s$  system should be highly deformed inside the charm-meson compared to conventional  $0^-(1^-)$  states. On the contrary, there is no any suppression rule on  $D_0^* \rightarrow D\pi$  or  $D'_1 \rightarrow D^*\pi$  so that the

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masses (widths) are not far away from our expectations. Therefore, it is important to find out a proper method to explain the observed states  $D_{sJ}^*$  in the framework of the quark model.

Recently, Deandrea *et. al.* [19] have showed that in terms of the constituent quark meson (CQM) model [20, 21], the masses of  $D_{sJ}^*$  could be constructed to be consistent with the measurements at  $B$  factories. It is interesting to question whether the CQM model can really address the other suffering problems in  $D_{sJ}^*$  mesons. In order to uncover the doubt, we calculate the decay constant of  $D_{s0}^*(2317)$ , defined by

$$\langle 0 | \bar{c} \gamma^\mu s | D_{s0}^*(2317) \rangle = i p^\mu f_{D_{s0}^*}, \quad (1)$$

in the CQM model and make a comparison to  $f_{D_s}$ . Under the heavy quark limit, we also assume  $f_{D_{s1}^*}(2460) \sim f_{D_{s0}^*}(2317)$ . Since the CQM model is based on the bosonized Nambu-Jona-Lasinio model and heavy quark symmetry, the constituent quark mass should be governed by the Schwinger-Dyson equation

$$\begin{aligned} m &= m_0 + 8mG_1 I_1, \\ I_1 &= \frac{iN_c}{16\pi^4} \int^{reg} d^4k \frac{1}{k^2 - m^2} = \frac{N_c m^2}{16\pi^2} \Gamma \left( -1, \frac{m^2}{\Lambda^2}, \frac{m^2}{\mu^2} \right), \end{aligned} \quad (2)$$

where  $G_1$  is taken to be as a parameter and flavor independent,  $m_0$  is the current quark mass,  $N_c = 3$  is the color number,  $\Lambda$  is the ultraviolet (UV) cutoff, which is the scale to separate the degrees of quark freedom from those at hadron level, and  $\mu$  is the infrared (IR) cutoff, which is used to interrupt the confining effects. Furthermore, the chosen values of  $\Lambda$  and  $\mu$  have to satisfy the condition  $\Delta = M - m_Q \geq m$  to guarantee no effects from confinement, in which  $M$  and  $m_Q$  correspond to the heavy meson and quark. At the heavy quark symmetry limit, it is known that the doublet  $(0^-, 1^-)$  can be related to  $(0^+, 1^+)$ , *i.e.*, one can obtain  $\Delta_S$  from  $\Delta_H$  where  $H(S)$  denote parity-odd (even) multiplet. Hence, according to the result of Ref. [21], the relationship for  $\Delta_H$  and  $\Delta_S$  is given by  $\Pi(\Delta_H) = \Pi(\Delta_S)$  with

$$\begin{aligned} \Pi(\Delta_{H,S}) &= I_1 + (\Delta_{H,S} \pm m) I_3(\Delta_S), \\ I_3(\Delta) &= \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \frac{e^{-s(m^2 - \Lambda^2)}}{s^{3/2}} (1 + \text{erf}(\Lambda\sqrt{s})), \end{aligned} \quad (3)$$

where the  $\Pi$  function comes from the meson self energy. Consequently, the renormalization constant of the meson could be obtained via  $Z^{-1}(x) = (d\Pi(x)/dx)_{x=\Delta}$  and described by

$$Z_{H,S}^{-1} = (\Delta_{H,S} \pm m) \frac{\partial I_3(\Delta_{H,S})}{\partial \Delta_{H,S}} + I_3(\Delta_{H,S}). \quad (4)$$

After introducing the basics formulas in the CQM model, the decay constant  $f_{D_{sJ}^*}(2317)$  can be calculated in terms of the definition in Eq. (1). To be more clear, the corresponding diagram is shown in Fig. 1. Hence, the explicit

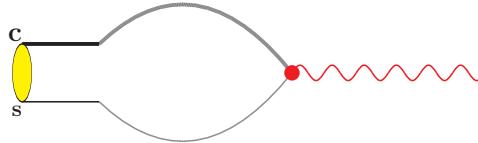


FIG. 1: Diagram for weak decay of  $D_{sJ}^*$

expression is written as [21]

$$f_{D_{s0}^*}(2317) = \frac{2}{\sqrt{m_{D_{s0}^*}}} \sqrt{Z_S} \Pi(\Delta_S). \quad (5)$$

To obtain the numerical value, we adopt  $G_1 = 5.25 \text{ GeV}^{-2}$ ,  $\mu = m = 0.5$  and  $\Lambda = 1.25 \text{ GeV}$  which are used in Ref. [19] to reach the right spectra of  $D_{sJ}^*$ . Immediately, we get  $f_{D_{s0}^*} \approx 176 \text{ MeV}$ . With the similar process, the decay constant of  $D_s$  is also estimated to be  $f_{D_s} \approx 270 \text{ MeV}$ . Is the obtained value of  $f_{D_{sJ}^*}$  in the CQM model proper although the value of  $f_{D_s}$  is still consistent with that in particle data group (PDG) [22]? We could examine this validity by the BELLE's observations in  $B$  decays. BELLE recently reports the decay branching ratios of  $B \rightarrow D_{sJ}^* \bar{D}$

to be [23]

$$\begin{aligned} BR(B^+ \rightarrow D_{sJ}^{*+}(2317)\bar{D}^0) \times BR(D_{sJ}^{*+} \rightarrow D_s^+\pi^0) &= (8.1_{-2.7}^{+3.0} \pm 2.4) \times 10^{-4}, \\ BR(B^+ \rightarrow D_{sJ}^{*+}(2317)\bar{D}^0) \times BR(D_{sJ}^{*+} \rightarrow D_s^{*+}\gamma) &= (2.5_{-1.6}^{+2.1})(< 7.6) \times 10^{-4}, \\ BR(B^+ \rightarrow D_{sJ}^{*+}(2460)\bar{D}^0) \times BR(D_{sJ}^{*+} \rightarrow D_s^{*+}\pi^0) &= (11.9_{-4.9}^{+6.1} \pm 3.6) \times 10^{-4}, \\ BR(B^+ \rightarrow D_{sJ}^{*+}(2460)\bar{D}^0) \times BR(D_{sJ}^{*+} \rightarrow D_s^{*+}\gamma) &= (5.6_{-1.5}^{+1.6} \pm 1.7) \times 10^{-4}, \\ BR(B^+ \rightarrow D_{sJ}^{*+}(2460)\bar{D}^0) \times BR(D_{sJ}^{*+} \rightarrow D_s^{*+}\gamma) &= (3.1_{-2.3}^{+2.7})(< 9.8) \times 10^{-4}. \end{aligned}$$

By assuming that  $D_{sJ}^*$  mainly decay to  $D^{(*)}\pi$  and  $D^{(*)}\gamma$  and taking the central values of measured results, the BRs of the decays  $B^+ \rightarrow D_{s0}^{*+}(2317)\bar{D}^0$  and  $B^+ \rightarrow D_{s1}^{*+}(2460)\bar{D}^0$  are roughly estimated to be  $10.6 \times 10^{-4}$  and  $20.6 \times 10^{-4}$ , respectively. Since the decays  $B^+ \rightarrow D_{sJ}^{*+}\bar{D}^0$  are color-allowed processes, the nonfactorizable effects are negligible. Therefore, comparing to the BRs of  $B^+ \rightarrow D_s^+\bar{D}^0$  and  $B^+ \rightarrow D_s^{*+}\bar{D}^0$  in PDG [22], we have

$$R_1 \equiv \frac{BR(B^+ \rightarrow D_{s0}^{*+}(2317)\bar{D}^0)}{BR(B^+ \rightarrow D_s^+\bar{D}^0)} = \frac{f_{D_{s0}^*}^2}{f_{D_s}^2} \sim 0.10, \quad (6)$$

$$R_2 \equiv \frac{BR(B^+ \rightarrow D_{s1}^{*+}(2460)\bar{D}^0)}{BR(B^+ \rightarrow D_s^{*+}\bar{D}^0)} = \frac{f_{D_{s1}^*}^2}{f_{D_s^*}^2} \sim 0.23. \quad (7)$$

With the previous obtained value of  $f_{D_{s0}^*} \approx 176$  MeV, we clearly see that  $R_{1CQM} \sim R_{2CQM} \sim 0.42$  are much bigger than experimental requirements. In order to reduce the values of  $f_{D_{sJ}^*}$ , it seems that we should adopt different values of the parameters  $\Delta_{H,S}$ ,  $\Lambda$  and  $\mu$ . For simplicity, we still fix  $\Lambda$  to be 1.25 GeV. The predicted results in the CQM model with various values of  $m_s$  and  $\mu$  by using  $\Delta_H = m_s$  are displayed in Table I. From the table, we find that if

TABLE I: Decay constants ( $f_{D_{s0}^*}, f_{D_s}$ ) with various values of  $m_s$  and  $\mu$  by fixing  $\Delta_H = m_s$ .

$\mu \setminus m_s$ (GeV)	0.5	0.55	0.6
0.35	(0.154, 0.262)	(0.154, 0.267)	(0.154, 0.272)
0.25	(0.123, 0.246)	(0.122, 0.250)	(0.122, 0.253)

we take  $\mu = 0.35$  GeV, the ratio defined in Eq. (6) will be  $R_{1,2} \sim 0.33$ . However, if we use  $\mu = \Lambda_{QCD} = 0.25$  GeV, the ratio  $R_{1,2CQM}$  could be around 0.24, which is still large for  $R_1$  although it fits  $R_2$  well. Nevertheless, according to the analysis of Ref. [19], the calculated  $\Delta_S - \Delta_H \sim 210$  MeV implies that the average mass of the parity-even meson, defined as  $M_S = (3M_{D_{s1}^*} + M_{D_{s0}^*})/4$ , is around 2286 MeV. By taking  $M_{D_{s0}^*} = 2317$  MeV, we get  $M_{D_{s1}^*} = 2275$  MeV which is even smaller than  $M_{D_{s0}^*}$ . It seems that if we just concentrate on the formulas of the heavy quark limit in the CQM model, the suffering problems, low mass and large decay constant, cannot be solved.

In summary, we have studied the spectra and decay constants of  $D_{sJ}^*$  in the CQM model simultaneously. We have shown that the chosen values of the parameters for fitting the corrected mass of  $D_{sJ}^*$  will give too big decay constants  $f_{D_{sJ}^*}$ , which are constrained by the observed decays  $B \rightarrow D_{sJ}^*\bar{D}$ . We note that the predicted decay constants on  $D_{sJ}^*(2317)$  and  $D_{sJ}^*(2460)$  are 84 and 126 MeV in the covariant light-front QCD [24], respectively. We conclude that the conventional approach is difficult to fit both spectra and decay constants of  $D_{sJ}^*$ .

### Acknowledgments

This work is supported in part by the National Science Council of R.O.C. under Grant No. NSC-91-2112-M-001-053 and No. NSC-92-2112-M-006-026.

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